**11.2 Simplifying Radical Expressions Date: \_\_\_\_\_\_\_\_\_\_**

**Establishing the Properties of Rational Exponents**

In the past, you have used properties of integer exponents to simplify and evaluate expressions. These properties are the same for rational exponents. Recall the properties below:

|  |
| --- |
| **Properties of Rational Exponents** |
| For all nonzero real numbers *a* and *b* and rational numbers *m* and *n*.

|  |  |  |
| --- | --- | --- |
| **Words** | **Algebra** | **Numbers** |
| **Product of Powers Property**To multiply powers with the same base, add the exponents. | $a^{m}∙a^{n}= \\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_$  | $12^{\frac{1}{2}}∙12^{\frac{3}{2}}=$  |
| **Quotient of Powers Property**To divide powers with the same base, subtract the exponents. | $\frac{a^{m}}{a^{n}}= \\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_$  | $\frac{125^{\frac{2}{3}}}{125^{\frac{1}{3}}}=$  |
| **Power of a Power Property**To raise one power to another, multiply the exponents. | $\left(a^{m}\right)^{n}=$ \_\_\_\_\_\_\_\_\_\_\_\_\_\_ | $\left(8^{\frac{2}{3}}\right)^{3}=$  |
| **Power of a Product Property**To find a power of a product, distribute the exponent. | $\left(ab\right)^{m}=\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_$  | $\left(16∙25\right)^{\frac{1}{2}}=$  |
| **Power of a Quotient Property**To find the power of a quotient, distribute the exponent. | $\left(\frac{a}{b}\right)^{m}= \\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_$  | $\left(\frac{16}{81}\right)^{\frac{1}{4}}=$  |

 |

**Simplifying Rational-Exponent Expressions**

***Learning Target C:*** *I can use the properties of rational exponents to simplify rational-exponent expressions.*

**Simplify the expression. Assume that all variables are positive. Exponents in simplified form should all be positive.**

**A)** $25^{\frac{3}{5}}∙25^{\frac{7}{5}}$ **B)** $\frac{8^{\frac{1}{3}}}{8^{\frac{2}{3}}}$

**C)** $\left(\frac{y^{\frac{4}{3}}}{16y^{\frac{2}{3}}}\right)^{\frac{3}{2}}$  **D)** $\left(27x^{\frac{3}{4}}\right)^{\frac{2}{3}}$

**E)** $\left(12^{\frac{2}{3}}∙12^{\frac{4}{3}}\right)^{\frac{3}{2}}$ **F)** $\frac{\left(6x^{\frac{1}{3}}\right)^{2}}{x^{\frac{5}{3}}y}$

**Simplifying Radical Expressions Using the Properties of Exponents**

***Learning Target D:*** *I can use the properties of exponents to simplify radical expressions.*

**Simplify the expression by writing it using rational exponents and then using the properties of rational exponents. Assume that all variables are positive. Exponents in simplified form should all be positive.**

**A)** $x\left(\sqrt[3]{2y}\right)\left(\sqrt[3]{4x^{2}y^{2}}\right)$ **B)** $\frac{\sqrt{64y}}{\sqrt[3]{64y}}$

**C)** $\frac{\sqrt{x^{3}}}{\sqrt[3]{x^{2}}}$ **D)** $\sqrt[6]{16^{3}}∙\sqrt[4]{4^{6}}∙\sqrt[3]{8^{2}}$

**Simplifying Radical Expressions Using the Properties of** $n^{th}$ **Roots**

***Learning Target E:*** *I can use the properties of* $n^{th}$ *to simplify radical expressions.*

From working with square roots, you know, for example that

$\sqrt{8}∙\sqrt{2}=\\_\\_\\_\\_\\_\\_=\\_\\_\\_\\_\\_\\_\\_\\_$ and $\frac{\sqrt{8}}{\sqrt{2}}=\\_\\_\\_\\_\\_\\_=\\_\\_\\_\\_\\_\\_\\_$

The corresponding properties also apply to $n^{th}$ roots.

|  |
| --- |
| **Properties of** $n^{th}$ **Roots** |
| For $a>0$ and $b>0$

|  |  |  |
| --- | --- | --- |
| **Words** | **Algebra** | **Numbers** |
| **Product Property of Roots**The $n^{th}$ root of a product is equal to the product of the $n^{th}$ roots. |  |  |
| **Quotient Property of Roots**The $n^{th}$ root of a Quotient is equal to the Quotient of the $n^{th}$ roots. |  |  |

 |

**Simplify the expression using the properties of** $n^{th}$ **roots. Assume that all variables are positive. Rationalize any irrational denominators.**

**A)** $\sqrt[3]{256x^{3}y^{7}}$ **B)** $\sqrt[4]{\frac{81}{x}}$

**C)** $\sqrt[3]{216x^{12}y^{15}}$ **D)** $\sqrt[4]{\frac{16}{x^{14}}}$

**Rewriting a Radical-Function Model**

***Learning Target F:*** *I can use the properties of rational exponents and properties of roots to rewrite radical function models and use these models to evaluate real-life situations.*

**When you find or apply a function model involving rational powers or radicals, you can use the properties of this lesson to help you find a simpler expression for the model.**

**A)** **Manufacturing.** A can that is twice as tall as its radius has the minimum surface area for the volume it contains. The formula $S=6π\left(\frac{V}{2π}\right)^{\frac{2}{3}}$ expresses the surface area of a can with this shape in terms of its volume.

**a.** Use the ***properties of rational exponents*** to simplify the expression for the surface area. Then write the approximate model with the coefficient rounded to the nearest hundredth.

**b.** Graph the model using a graphing calculator. What is the surface area in square centimeters for a can with a volume of 440 $m^{3}$ ?

**B) Commercial Fishing.** The buoyancy of a fishing float in water depends on the volume of air it contains. The radius of a spherical float as a function of its volume is given by $r=\sqrt[3]{\frac{3V}{4π}}$.

**a.** Use the ***properties of roots*** to rewrite the expression for the radius as the product of a coefficient term and a variable term. Then write the approximate formula with the coefficient rounded to the nearest hundredth.

**b.** What should the radius be for a float that needs to contain 4.4 $ft^{3}$ of air to have the proper buoyancy?